# A rain field reconstruction procedure based on multisensor data for storm analysis and prediction in the Mediterranean

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## ABSTRACT

The proposed methodology produces stochastic rainfall fields at various space-time scales, based on the reconstruction of large scale precipitation patterns that are obtained from multi-sensor observations. These can be interpreted indeed as an estimate of the average precipitation for specified aggregates of data taken over a suitable operational resolution and do not directly capture the intrinsic variability of the rainfall process at scales that are finer than the sensor footprint at the ground. A series of rainfall fields at decreasing space-time scales are reproduced with a constrain to the available rain gauge data. The algorithm maximizes joint probabilities in non-constrained nodes, conditional on the specified values at constraining nodes and on the base field, by operating on the covariance structure. Each generated field is associated with a reliability map, which measures the residual variance due to the estimation uncertainty. The case study of an observed precipitation event over northwest Italy is examined and results are discussed.

# 1 INTRODUCTION

Whenever a physical phenomenon is observed at a very aggregate scale its small-scale features are unavoidably lost (Bacchi and Kottegoda, 1995). In particular, this is a problem that has to be faced when the rainfall field is reconstructed for use as input to the simulation of the hydrological response of small to medium size basins, which requires adequate representation scales at fine resolution.

The actually available sensors for rainfall monitoring are characterized by quite different scales of spatial and temporal resolution. In addition, the measures are relevant to different physical variables, and the reliability of the derived rainfall estimates varies depending on the sensor (Lanza *et al.*, 1997).

The present work aims at suitable integration of data coming from the interpretation of Meteosat infrared (IR) images and rain-gauge measurements at the ground. We assume that Meteosat data are relevant to the stochastic characterization of the rainfall field only, whereas rain-gauge measurements are considered as the ground truth at point scale. More specifically, Meteosat IR images are used to infer the mean and variance of the random variable used to describe the rainfall field. Besides, the joint distribution of such variables is a priori selected.

The proposed procedure allows reconstruction of a stochastic rainfall field based on a joint probability distribution conditional to the real measurement values given by the rain gauges. The correlation structure of such a rainfall field is that estimated by the Meteosat sensor at the coarse scale.

# 2 THE PROPOSED PROCEDURE

Let us consider a region where N rain-gauge measurement points are available, as well as Meteosat data $x_k$, $k = 1,....,m \cdot n$, defined on a regular grid. The desired scale for reconstruction of the rainfall field is that of the Meteosat pixels (about 7x5 km$^2$ at mid-latitude). For the sake of simplicity, it is assumed that at most one single rain-gauge is located in each pixel, and that its measure may be reduced by means of a proper area reduction factor (ARF) to the area average value over the pixel size (Rodriguez-Iturbe and Mejìa, 1974).

The following further assumptions are made:

- the joint probability distribution of the log rainfall field variables $z_k$ (each one referred to a pixel), k=1,…, $m \cdot n$, is Gaussian;
- the mean and variance of $z_k$ as a function of the Meteosat measurement $x_k$, may be obtained in the form

$$\mu(z_k) = f(x_k) \qquad \sigma^2(z_k) = g(x_k);$$

- the rain-gauge measurements $w_k$ are retained as realizations of $z_k$;

• $\rho_{ij}$ is the correlation between pixels i and j; the field is assumed isotropic and space-invariant (i.e., $\rho_{ij}$ depends only from the distance between pixels i and j).

The stochastic random field can then be characterized by

$$Z = (Z_1, Z_2) = (z_1, z_2, ..., z_N, z_{N+1}, ...., z_{m \cdot n}) \quad N << m \cdot n \tag{1}$$

with average

$$\underline{\mu} = [\mu_1, ...., \mu_N, \mu_{N+1}, ..., \mu_{m \cdot n}]^T \tag{2}$$

and covariance matrix

$$\Sigma = \begin{bmatrix} \sigma^2_{z_1} & . & \sigma^2_{z_1, z_N} & \sigma^2_{z_1, z_{N+1}} & . & \sigma^2_{z_1, z_{m \cdot n}} \\ . & . & . & . & . & . \\ \sigma^2_{z_N, z_1} & . & \sigma^2_{z_N} & \sigma^2_{z_N, z_{N+1}} & . & \sigma^2_{z_N, z_{m \cdot n}} \\ \sigma^2_{z_{N+1}, z_1} & . & \sigma^2_{z_{N+1}, z_N} & \sigma^2_{z_{N+1}} & . & \sigma^2_{z_{N+1}, z_{m \cdot n}} \\ . & . & . & . & . & . \\ \sigma^2_{z_{m \cdot n}, z_1} & . & \sigma^2_{z_{m \cdot n}, z_N} & \sigma^2_{z_{m \cdot n}, z_{N+1}} & . & \sigma^2_{z_{m \cdot n}} \end{bmatrix} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \tag{3}$$

where

$$\sigma^2_{z_i z_j} = \rho_{i,i} \cdot \sigma_{z_i} \cdot \sigma_{z_j} \tag{4}$$

The joint probability density function conditional on the N measurements of the rain gauges is (Box *at al.*, 1994):

$$p(Z_2 | Z_1) = (2\pi)^{-(m \cdot n - N)} \cdot |\Sigma_{22.11}|^{-\frac{1}{2}} \cdot \exp\left[ -\frac{(Z_2 - \underline{\mu}_{2.1})^t \cdot \Sigma^{-1}_{22.11} \cdot (Z_2 - \underline{\mu}_{2.1})}{2} \right] \tag{5}$$

where

$$\underline{\mu}_{2.1} = \underline{\mu}_2 + \beta_{2.1} \cdot (\overline{Z}_1 - \underline{\mu}_1) = E(Z_2 | Z_1) \tag{6}$$

$$\Sigma_{22.11} = \Sigma_{22} - \beta_{2.1} \cdot \Sigma_{12} \tag{7}$$

$$\beta_{2.1} = \Sigma^t_{12} \cdot \Sigma^{-1}_{11} \tag{8}$$

and $\overline{Z}1 = (w_1,....,w_N)$.

Such a probability density function is maximized by choosing $\mu_{2.1} = E(Z_1, Z_2)$, which is then selected as the rainfall field estimation procedure.

## 2.1 Reliability maps of the reconstructed rainfall field

The reliability of the reconstructed rainfall field can be characterized by suitable maps as follows. Let us compute the variance of the error of estimator $\mu_{2.1}$.

It is known that:

$$\sigma^2_{err_{z_i^*}} = E\left[(z_i - z_i^*)^2\right] \tag{9}$$

The estimator is defined as:

$$z_i^* = \mu_i + \sum_{j=1}^{N} \beta_j \cdot (z_j - \mu_j) \tag{10}$$

Let us introduce a new random variable y defined as:

$$y_i^* = z_i^* - \mu_i \tag{11}$$

$$y_j = z_j - \mu_j \tag{12}$$

The variance of the error for a generic variable $y_i$ is:

$$\sigma^2_{err_{z_i^*}} = \sigma_i^2 + \sum_{l=1}^{N}\sum_{m=1}^{N} \beta_l \beta_m \Sigma_{11} - 2\sum_{j=1}^{N} \beta_j \Sigma_{12} \tag{13}$$

Then, the reliability of the estimate for pixel i expressed by its explained variance, can be evaluated through:

$$\sigma^2_{\exp} = 1 - \frac{\sigma^2_{err_{z_i^*}}}{\sigma_i^2 + \sigma_M^2} \tag{14}$$

where $\sigma_M^2$ represents the variance of the whole Meteosat field.

# 3 CASE STUDY

The case study presented here refers to the storm observed on 28 May, 1998 over a spatial region of about 25000 $Km^2$ in Northern Italy, by the 34 active rain-gauges available in the area. The rainfall process has been observed from 12:00 a.m. to 14:00 p.m. In Fig. 1 a sample Meteosat image of the event at 12:00 UT is presented, so as to picture the meteorological situation at synoptic scale.

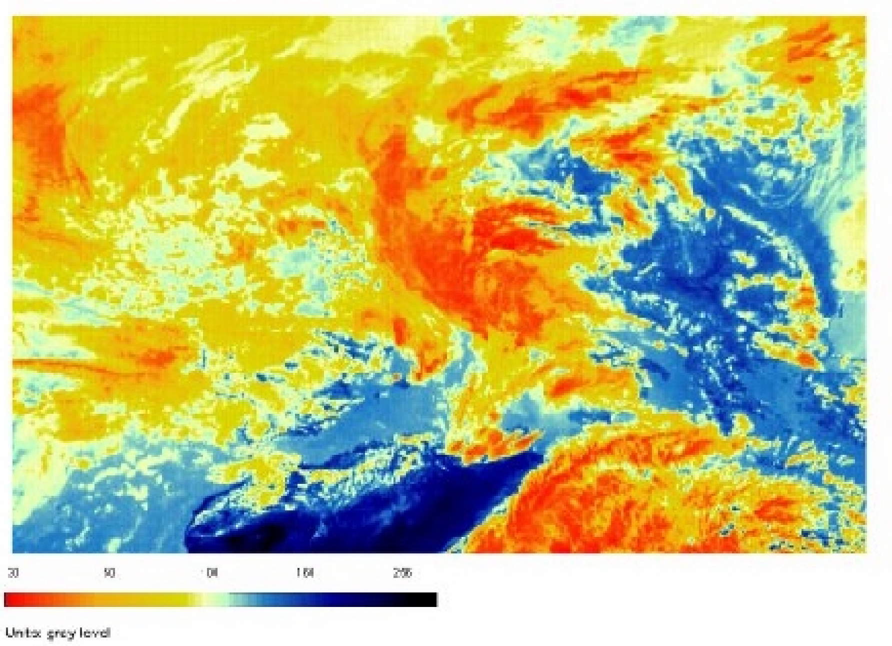

Figure 1: Meteosat image.

The function expressing the mapping from the Meteosat measurement (for a single pixel) to the mean value of *z* is plotted in Fig. 2, transforming the relation of Adler and Negri (1988). For setting the variance a constant CV has been assumed equal to 1. A gaussian model has been selected for the spatial correlation function (see Fig. 3).

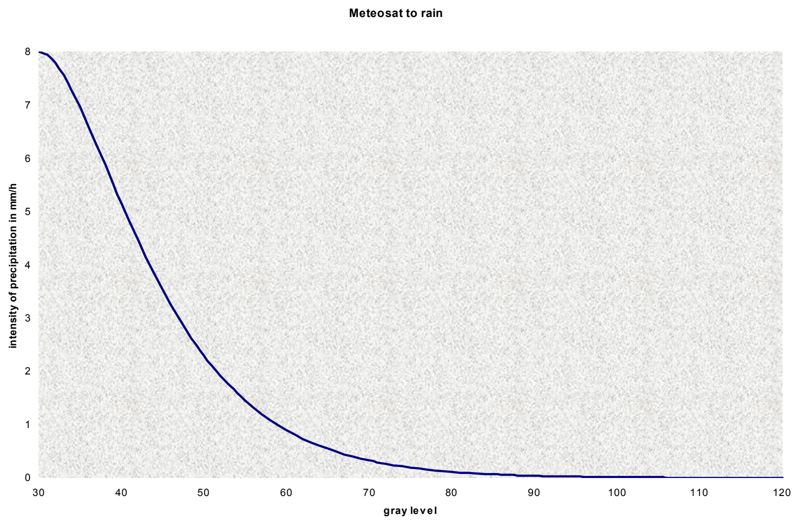


Figure 2: mapping Meteosat data to rainfall data

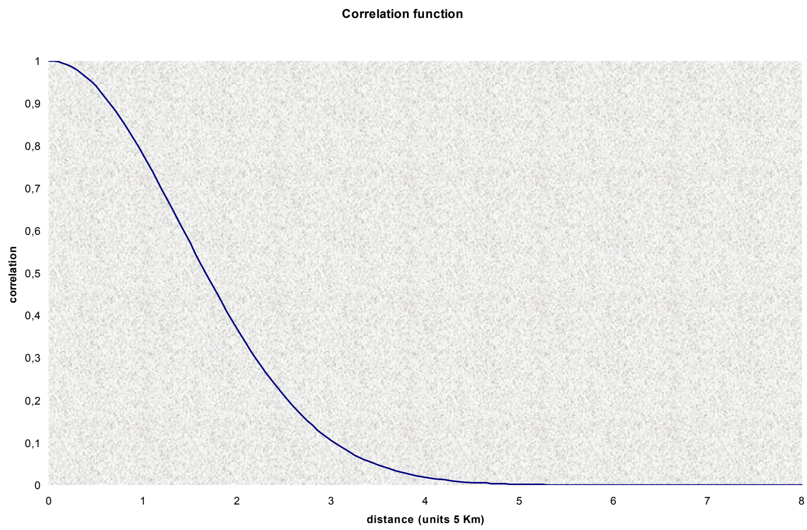


Figure 3: spatial correlation function

The rainfall field obtained by integrating the data coming from the two sensors and the map of the standard deviations of the random variables of the reconstructed field are shown in Figs. 4 and 5. Besides, in Fig. 6 the estimate reliability map is plotted.

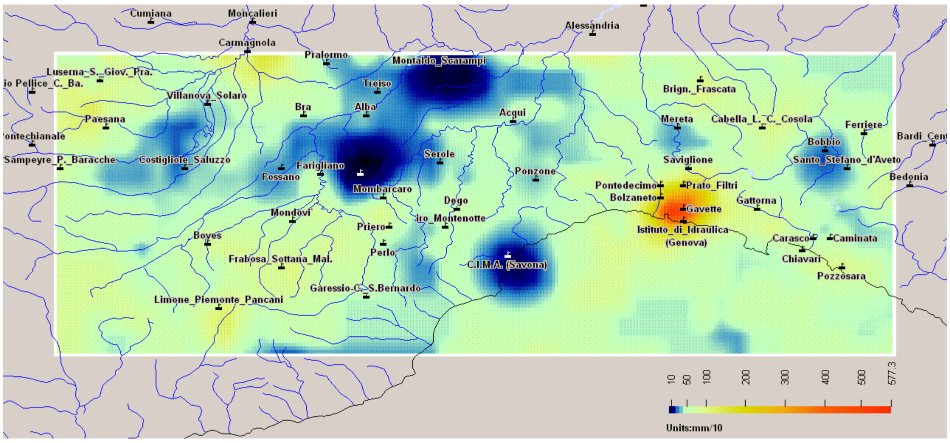


Figure 4: Rainfall field from Meteosat data constrained to rain-gauge (2 h cumulated)

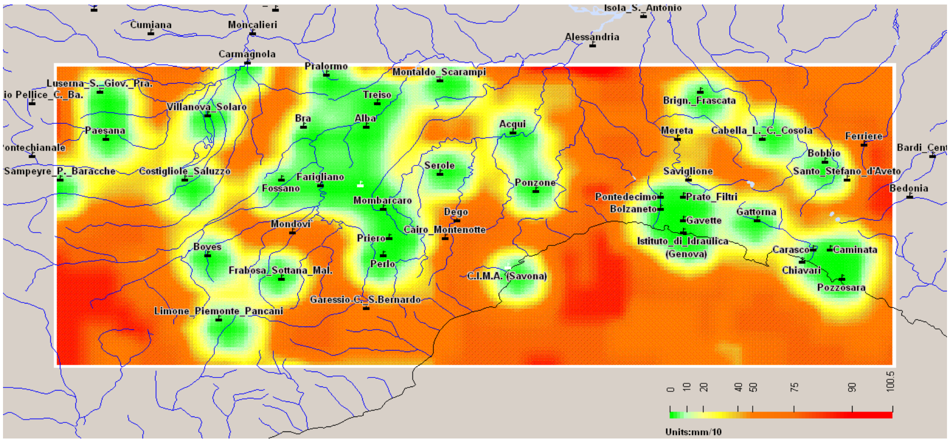


Figure 5: Standard deviation of rain fluctuations with respect to the Meteosat field constrained to rain-gauge (given CV = cost)

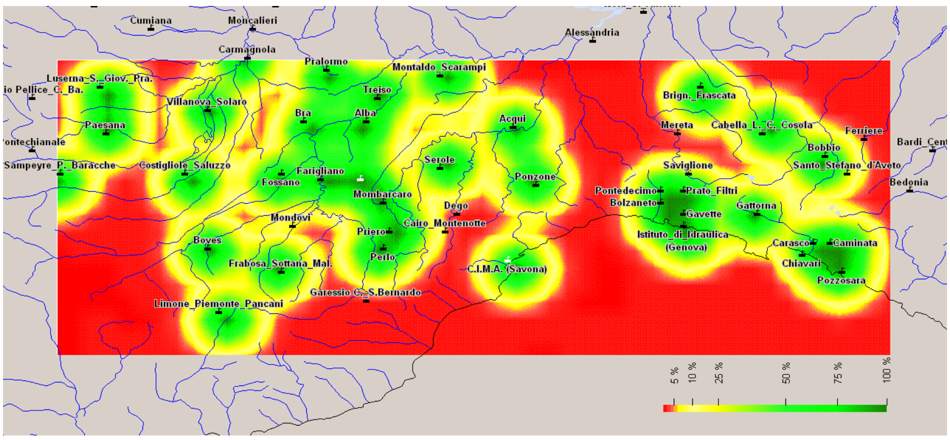


Figure 6: Reliability map of the rain field

The rainfall field in Fig. 4 has been resized with a finer grid and the mean value has been used for determination of the parameters of the new joint probability density function at the finer scale. The same procedure is than used for determination of the rainfall field at a finer scale, constrained at point values at the corresponding new time scale.

Fig. 7 and Fig. 8 show the rainfall field obtained. In Fig. 9 the map of the standard deviation of the random variables of the reconstructed fields is shown. In Fig. 10 the corresponding reliability map is plotted.

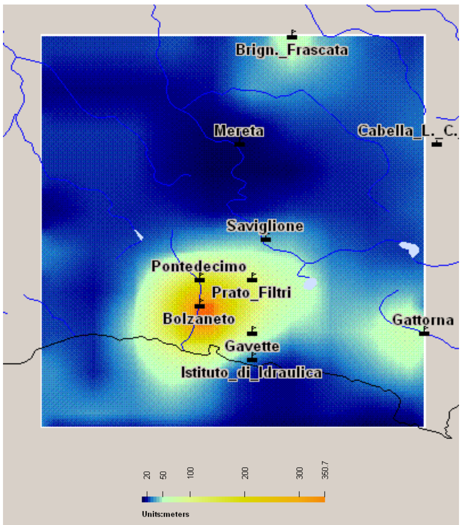


Figure 7:
Disaggregated rainfall field (first hour)

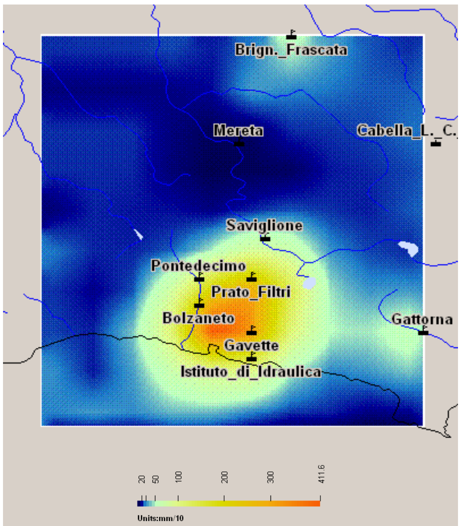


Figure 8:
Disaggregated rainfall field (second hour)

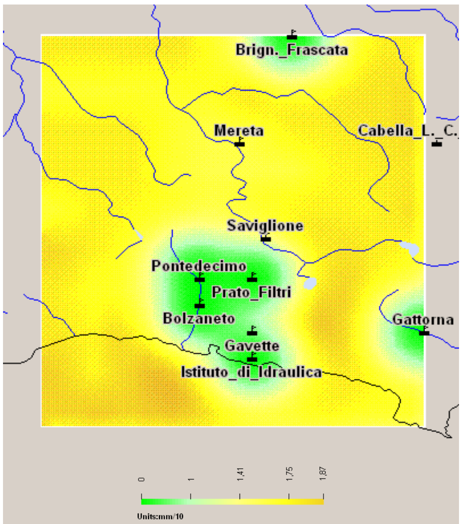


Figure 9:
St. dev. of disaggregated rainfall fields

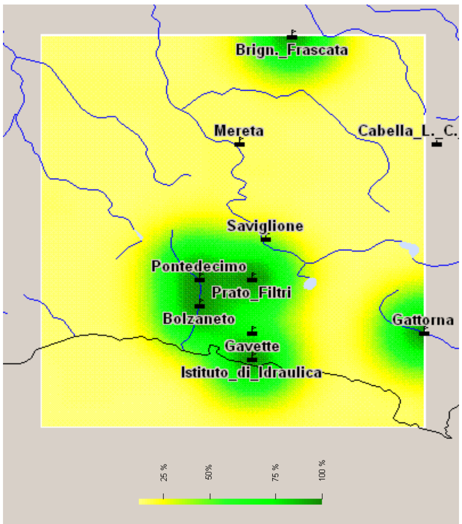


Figure 10:
Reliability map of disaggregated fields

# 4 VALIDATION

In order to assess the overall reliability of the reconstructed rain field, the following steps have been addressed:

1) N rainfall fields have been reconstructed (as many as the number of active rain gauges). Each of them has been obtained by neglecting the information coming from a specific rain gauge in the reconstruction procedure;

2) for each of such fields, the rainfall value obtained at the location corresponding to the "neglected" rain-gauge has been compared with the actual value measured by the rain gauge;

3) the correlation $\rho_1$ between the above set of values, and the correlation $\rho_2$ between the Meteosat derived data and the real measurements, have been computed (see Fig. 11, 12).

Such a validation procedure provided quite satisfactory results. More specifically, $\rho_1$ has been found to equal 0,71, much higher than the value 0,23 obtained for coefficient $\rho_2$.

Actually, a high correlation $\rho_1$ would indicate a certain redundancy of the number of measuring instruments, whereas a low correlation could indicate that rain-gauge locations are too sparsely distributed.

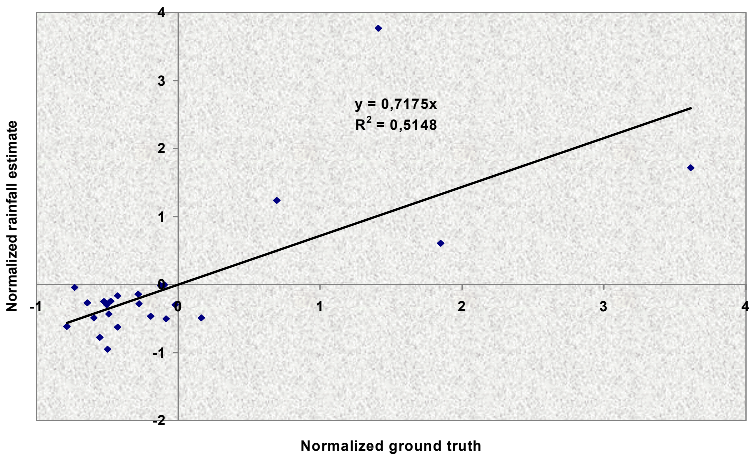


Figure 11: Scatterplot between rainfall estimate and observed rainfall

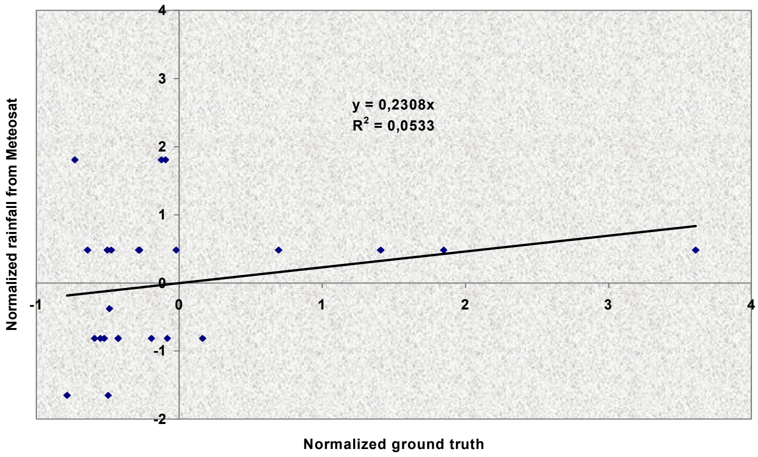


Figure 12: Scatterplot between Meteosat data and observed rainfall

# 5 CONCLUSIONS

A procedure has been proposed to integrate rainfall information coming from different sources. Each of them provides information which is different in terms of reliability and scale. The procedure presented in this paper allows to obtain the following results:

1) the expected values of the reconstructed rainfall field at the same scale of the available area information using point data as a constraint;

2) the residual variance of the precipitation process and therefore a measure of the information content of the estimate field;

3) a quantitative evaluation (in percentage terms) of the reliability of the rainfall field in every cell of the grid.

On the basis of the latter evaluation, it is possible to define a procedure to assess the quality of the distribution of the monitoring network.

The probabilistic information obtained for the rainfall field can be used for the stochastic generation of different scenarios, with the purpose of determining the risk levels of specific areas in the considered region, conditional on the available information of the specific rainfall event.

The possibility of determining the rainfall field at a finer scale than the original one has been also considered for disaggregation purposes.

The procedure developed, for the reconstruction of the rainfall field, is based on the definition of various functions, which are used for conversion of the original information (as provided by the available sensors) into quantitative information about rainfall intensity, or better into suitable parameters characterizing the probabilistic distribution of the rainfall field. Of course, validation of such functions is needed by means of an extensive comparison with actual rainfall measurements.

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